KASR TARTIBLI DIFFERENSIAL VA INTEGRO- DIFFERENSIAL TENGLAMALAR UCHUN NOLOKAL MASALALAR

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Annotatsiya: *Ushbu maqolada Kaputo kasr tartibli differensial operator va Prabhakar kasr tartibli integral operatori qatnashgan integro-differensial tenglama hamda Prabhakar differensial operatori qatnashgan kasr tartibli tenglama uchun nolokal shartli masalalarning yaqqol ko`rinishdagi yechimlari topilgan.*

Kalit so`zlar: *Kasr tartibli differensial tenglamalar; Kaputo operatori; Prabhakar operatori.*

NONLOCAL PROBLEMS FOR FRACTIONAL ORDER DIFFERENTIAL AND INTEGRO-DIFFERENTIAL EQUATIONS

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Abstract: *In this paper solutions of nonlocal problems for fractional order integro-differential equations with the Caputo and Prabhakar operators have been found explicitly.*

Keywords: *fractional differential equations; Caputo operator; Prabhakar operator.*

Kasr tartibli differensial tenglamalar fizika, mexanika, iqtisod, ximiya va ko`plab boshqa soxalarning amaliy masalalarni matematik modellashtirishda ishlatilmoqda $[1]$. Bunday tenglamalar uchun o`rganiladigan masalalardan asosiylari boshlang`ich shartli masala (Koshi masalasi), chegaraviy masalalardi $[2]$. Bunday masalalarni yechishning Laplas almashtirishi, operator usuli va integral tenglamaga keltirib yechish usullari ko`p qo`llaniladi $[3]$.

Dastlab ushbu integro-differensial tenglamani $t>0$ da tadqiq etamiz:

$$ {}_C D_{0+}^{\mu} y(t) - \lambda \mathcal{E}_{0+,\alpha,\beta}^{\delta,\gamma,1} y(t) = f(t) \qquad (1)$$

Bu yerda $0<\mu<1$, $\delta\in\mathbb{R}$, $\alpha>0$, $\beta>0$, $\gamma>0$, $\lambda\in\mathbb{R}$, $f(t)-$berilgan funksiya,

$$ {}_C D_{0+}^{\mu} y(t) = \frac{1}{\Gamma(1-\mu)} \int_0^t \frac{y^{(1)}(z)dz}{(t-z)^{\mu}}, \quad 0<\mu\le 1 \qquad (2)$$

$-\mu$ kasr tartibli Kaputo differensial operatori $[3]$,

$$\mathcal{E}^{\delta,\gamma,\xi}_{0+,\alpha,\beta}y(t)=\int_a^t(t-z)^{\beta-1}\mathrm{E}^{\gamma,\xi}_{\alpha,\beta}\left[\delta(t-z)^{\alpha}\right]y(z)dz \qquad (3)$$

–umumlashgan Prabhakar integral operatori $[4]$.

$$\mathrm{E}^{\gamma,\xi}_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_{\xi n}}{\Gamma(\alpha n+\beta)}\cdot\frac{z^n}{n!} \qquad (4)$$

–Prabhakar funksiyasining bir umumlashmasi $[5]$.

1-masala. (1) tenglamaning

$$y(0)=\sigma y(t_0)\ ,\ \sigma\in\mathbb{R}/\{0\},\ t_0\in\mathbb{R}^+ \qquad (5)$$

shartni qanoatlantiruvchi yechimi topilsin.

Dastlab (1) tenglamaning $y(0)=A$, $A\in\mathbb{R}$ shartni qanoatlantiruvchi yechimini yozib olamiz $[4]$:

$$y(t)=A+\lambda t^{\mu+\beta}\mathrm{E}^{\gamma}_{\alpha,\beta+\mu+1}(\delta t^{\alpha})+\frac{1}{\Gamma(\mu)}\int_0^t(t-z)^{\mu-1}f(z)dz\,. \qquad (6)$$

So’ngra (6) yechimni (5) shartga bo`ysundiramiz:

$$\frac{A}{\sigma}=A+\lambda t_0^{\ \mu+\beta}\mathrm{E}^{\gamma}_{\alpha,\beta+\mu+1}(\delta t_0^{\ \alpha})+\frac{1}{\Gamma(\mu)}\int_0^{t_0}(t_0-z)^{\mu-1}f(z)dz\,. \qquad (7)$$

Qayd etish kerakki, agar $\sigma=0$ bo’lsa (5) nolokal shart $y(0)=0$ ga o`tib qolib, $(1)$ tenglamaning ushbu shartni qanoatlantiruvchi yechimi $(6)$ dan kelib chiqadi. Demak, $\sigma\neq1$ holda $(7)$ dan noma`lum A uchun

$$A=\frac{\sigma}{1-\sigma}\left[\lambda t_0^{\ \mu+\beta}\mathrm{E}^{\gamma}_{\alpha,\beta+\mu+1}(\delta t_0^{\ \alpha})+\frac{1}{\Gamma(\mu)}\int_0^{t_0}(t_0-z)^{\mu-1}f(z)dz\right] \qquad (8)$$

ifodani olamiz.

(6) da A ning o`rniga $(8)$ ifodani qo`ysak, (1), (5) masalaning yechimi quyidagi ko`rinishda yoziladi:

$$y(t)=\frac{\sigma}{1-\sigma}\left[\lambda t_0^{\ \mu+\beta}\mathrm{E}^{\gamma}_{\alpha,\beta+\mu+1}(\delta t_0^{\ \alpha})+\frac{1}{\Gamma(\mu)}\int_0^{t_0}(t_0-z)^{\mu-1}f(z)dz\right]+$$

$$+\lambda t^{\mu+\beta}\mathrm{E}^{\gamma}_{\alpha,\beta+\mu+1}(\delta t^{\alpha})+\frac{1}{\Gamma(\mu)}\int_0^t(t-z)^{\mu-1}f(z)dz \qquad (9)$$

Shunday qilib, quyidagi tasdiq o`rinli ekanligi isbotlandi.

1-teorema. Agar $\sigma\neq1$ va $f(t)\in AC^1[0,\infty)$ bo`lsa, u holda 1-masalaning yagona yechimi mavjud va u (9) formula orqali aniqlanadi.

Bu yerda AC^1 funksiyalar sinfi 1- tartibli hosilasi absolyut uzluksiz bo’lgan funksiyalarni o`z ichiga oladi $[3]$.

Endi quyidagi kasr tartibli differensial tenglamani

$$ {}^{PC}D_{0+}^{\alpha,\beta,\gamma,\delta}y(t)-\lambda y(t)=f(t) \qquad (10)$$

$t\in[0,1]$ kesmada tadqiq etamiz. Bu yerda $\lambda,\alpha,\beta,\gamma,\delta\in\mathbb{R}$, $1<\beta\le 2$, $f(t)$ – berilgan funksiya,

$$ {}^{PC}D_{0+}^{\alpha,\beta,\gamma,\delta}y(t)={}^{P}\mathrm{I}_{0+}^{\alpha,m-\beta,-\gamma,\delta}\frac{d^m}{dt^m}y(t), \quad m-1<\beta\le m$$

$-\beta$ kasr tartibli Kaputo ma`nosidagi Prabhakar kasr tartibli hosilasi $[4]$.

$$ {}^{P}\mathrm{I}_{0+}^{\alpha,\beta,\gamma,\delta}y(t)=\int_0^t (t-z)^{\beta-1}E_{\alpha,\beta}^{\gamma}(\delta(t-z)^{\alpha})f(z)dz$$

$-\beta$ kasr tartibli Prabhakar integrali $[6]$.

2-masala. (10) tenglamaning

$$ y(0)=a,\ \ y(1)=b,\ \ a,b\in\mathbb{R} \qquad (11)$$

shartlarni qanoatlantiruvchi yechimi topilsin.

Dastlab (10) tenglamaning $y(0)=A_1$, $y^{(1)}(0)=A_2$ boshlang`ich shartlarni qanoatlantiruvchi yechimini yozib olamiz .$[7]$ da bu masalaning yechismi

$$ y(t)=A_1+A_2t+\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=1}^{2}\frac{((1+j)\gamma)_j}{j!}\cdot\frac{A_k\lambda^{i+1}\delta^j t^{\alpha j(1+j)\beta+k}}{\Gamma(\alpha j+(i+1)\beta+k)}+$$

$$ +\int_0^t\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\frac{((1+j)\gamma)_j}{j!}\frac{\lambda^i\delta^j(t-z)^{\alpha j(1+j)\beta-1}}{\Gamma(\alpha j+(i+1)\beta)}f(z)dz \qquad (12)$$

ko`rinishda berilgan. $[8]$da esa ikki o`zgaruvchili Mittag-Leffler tipidagi funksiyadan foydalanib (12) ifoda quyidagi ko`rinishda yozilgan.

$$ y(t)=A_1+A_2t+\sum_{k=1}^{2}A_k t^{\beta+k-1}\Gamma(\gamma)\mathrm{E}_2\left(\begin{matrix}\gamma,\gamma,1;1,0\\ \beta+k,\beta,\alpha;\gamma,\gamma;1,1\end{matrix}\middle|\begin{matrix}\lambda t^{\beta}\\ \delta t^{\alpha}\end{matrix}\right)+$$

$$ +\int_0^t(t-z)^{\beta-1}\Gamma(\gamma)\mathrm{E}_2\left(\begin{matrix}\gamma,\gamma,1;1,0\\ \beta,\beta,\alpha;\gamma,\gamma;1,1\end{matrix}\middle|\begin{matrix}\lambda(t-z)^{\beta}\\ \delta(t-z)^{\alpha}\end{matrix}\right)f(z)dz \qquad (13)$$

Bu yerda

$$ \mathrm{E}_2\left(\begin{matrix}\gamma_1,\alpha_1,\beta_1;\gamma_2\alpha_2\\ \delta_1,\alpha_3,\beta_2;\delta_2,\alpha_4;\delta_3,\beta_3\end{matrix}\middle|\begin{matrix}x\\ y\end{matrix}\right)=\sum_{i,j=0}^{\infty}\frac{(\gamma_1)\alpha_1 i+\beta_1 j^{(\gamma_2)}\alpha_2 i}{\Gamma(\delta_1+\alpha_3 i+\beta_2 j)}\cdot\frac{x^i}{\Gamma(\delta_2+\alpha_4 i)}\cdot\frac{y^j}{\Gamma(\delta_3+\beta_3 j)}$$

– 2 o`zgaruvchili Mittag-Leffler tipidagi funksiya $[9]$.

(13) ni (11) shartga qo'ysak $A_1=a$ va

$$ A_2=\left[1+\Gamma(\gamma)\mathrm{E}_2\left(\begin{matrix}\gamma,\gamma,1;1,0\\ \beta+2,\beta,\alpha;\gamma,\gamma;1,1\end{matrix}\middle|\begin{matrix}\lambda\\ \delta\end{matrix}\right)\right]^{-1}\left(b-a-a\Gamma(\gamma)\mathrm{E}_2\left(\begin{matrix}\gamma,\gamma,1;1,0\\ \beta+1,\beta,\alpha;\gamma,\gamma;1,1\end{matrix}\middle|\begin{matrix}\lambda\\ \delta\end{matrix}\right)-\right.$$

$$ \left.-\int_0^1(1-z)^{\beta-1}\Gamma(\gamma)\mathrm{E}_2\left(\begin{matrix}\gamma,\gamma,1;1,0\\ \beta,\beta,\alpha;\gamma,\gamma;1,1\end{matrix}\middle|\begin{matrix}\lambda(t-z)^{\beta}\\ \delta(t-z)^{\alpha}\end{matrix}\right)f(z)dz\right)$$

ni topamiz.

Demak, 2-masalaning bir qiymatli yechimiga oid quyidagi tasdiq o`rinli.

2-teorema. Agar $f(t) \in C_\nu^{\ 2}[0,1]$ va

$$1+\Gamma(\gamma)\mathrm{E}_2\left(\begin{matrix}\gamma,\gamma,1;1,0\\ \beta+2,\beta,\alpha;\gamma,\gamma;1,1\end{matrix}\middle|\begin{matrix}\lambda\\ \delta\end{matrix}\right)\neq 0$$

bo`lsa, u holda 2-masalaning yagona yechimi mavjud va u (13) formula bilan aniqlanadi.

Bu yerda $C_\nu^{\ m}[0,1]$ funksiyalar sinfi $m-$tartibli hosilagacha $C_\nu[0,1]$ sinfga tegishli bo`lgan funksiyalarni o`z ichiga oladi. O`z navbatida C_ν sinf

$$f(x)=x^p f_1(x),\quad f_1\in C[0,1],\quad p>\nu,\quad (\nu>-1)$$

ko`rinishdagi funksiyalarni o`z ichiga oladi [3].

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