YUKLANGAN KASR TARTIBLI ODDIY DIFFERENSIAL TENGLAMALAR UCHUN MASALALAR

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Annotatsiya: *Ushbu maqolada Hilfer ma'nosidagi kasr tartibli operator qatnashgan yuklangan tenglama uchun ikkita masalani o`rganilgan. Bu masalalar yechimlari Koshi masalasi yechimidan foydalanib topilgan.*

Kalit so`zlar: *yuklangan oddiy differensial tenglama, kasr tartibli operator, Koshi masalasi.*

ЗАДАЧИ ДЛЯ ЗАГРУЖЕННЫХ ОБЫКНОВЕННЫХ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ ДРОБНОГО ПОРЯДКА

Аннотация: *В данной статье изучались две задачи для нагруженного уравнения с дробным оператором по Нильферу. Найдены решения этих задач с использованием решения задачи Коши.*

Ключевые слова: *нагруженное обыкновенное дифференциальное уравнение, оператор дробного порядка, задача Коши.*

PROBLEMS FOR LOADED ORDINARY DIFFERENTIAL EQUATIONS OF FRACTIONAL ORDER

Abstract: *In this article, two problems for a loaded equation with a fractional operator in the sense of Hilfer were studied. The solutions to these problems by using the solution of the Cauchy problem were found.*

Keywords: *loaded ordinary differential equation, fractional order operator, Cauchy problem.*

So'ngi vaqtlarda noma`lum funksiyani biror qiymati qatnashgan differensial tengalamalar bilan shug'ullanishga bo`lgan qiziqish ortib bormoqda. Bunga sabab ko`plab issiqlik tarqalishi va diffuziya jarayonlarini matematik modelini tuzish funksiyani biror qiymati qatnashgan differensial tenglama uchun qo'yiladigan masalalarga keltiriladi. Odatda, bunday turdagi tenglamalar yuklangan differensial tenglama deb yuritiladi. Yuklangan xususiy hosilali va oddiy differensial tenglamalar yuklangan differensial tenglama ko`plab tadqiqotchilar tomonidan o'rganilgan (masalan, ushbu [1]–[3] ishlarga qaralsin).

(0,1) oraliqda ushbu

$$D_{0x}^{\alpha,\beta} y(x) - \lambda y(x) = y(x_0) \qquad (1)$$

yuklangan kasr tartibli oddiy differensial tenglamani qaraylik,bu yerda $y(x)$-noma'lum funksiya; $\alpha, \beta, \lambda, x_0$-o'zgarmas haqiqiy sonlar bo'lib, $0<\alpha<1, 0\le\beta\le1, 0<x_0<1$; $D_{0x}^{\alpha,\beta} y(x)$ – Hilfer ma'nosidagi kasr tartibli hosila bo'lib,

$$D_{0x}^{\alpha,\beta}y(x)=I_{0x}^{\beta(1-\alpha)}\frac{d}{dx}I_{0x}^{(1-\beta)(1-\alpha)}y(x),\qquad(2)$$

$I_{0x}^{\gamma}y(x)$ – Riman-Liuvill ma'nosida γ (kasr) tartibli integral:

$$I_{0x}^{\gamma}y(x)=\frac{1}{\Gamma(\alpha)}\int_0^x(x-t)^{\gamma-1}y(t)dt,\gamma>0,\ I_{0x}^{\gamma}y(x)=\frac{1}{\Gamma(\gamma)}\int_0^x(x-t)^{\gamma-1}y(t)dt,\ \gamma>0.$$

A **masala.** Shunday $y(x)$ funksiya topilsinki, u quyidagi xossalarga ega bo'lsin:

1) $x^{(1-\beta)(1-\alpha)}y(x)\in C[0,1]$, $D_{0x}^{\alpha,\beta}y(x)\in C(0,1)$ sinfga tegishli

2) $y(x)$ funksiya (1) tenlamani qanoatlantirsin;

3)

$$\lim_{x\to0}x^{(1-\alpha)(1-\beta)}y(x)=A,\qquad(3)$$

bu yerda, A - berilgan o'zgarmas haqiqiy son.

Ma'lumki [4],

$$D_{0x}^{\alpha,\beta}y(x)-\lambda y(x)=f(x)$$

tenglamaning (3) shartni bajaruvchi yechimi

$$y(x)=Ax^{-(1-\alpha)(1-\beta)}E_{\alpha,1-(1-\alpha)(1-\beta)}(\lambda x^{\alpha})+\int_0^x(x-t)^{\alpha-1}E_{\alpha,\alpha}\left[\lambda(x-t)^{\alpha}\right]f(t)dt\qquad(4)$$

ko'rinishda bo'ladi, bu yerda $E_{\gamma,\sigma}(z)=\sum_{k=0}^{+\infty}\frac{z^k}{\Gamma(\gamma k+\sigma)}$ - Mittag-Leffler funksiyasi.

Bizning masalada $f(x)=y(x_0)$ bo'lgani uchun (1) tenglamaning (3) shartni qanoatlantiruvchi yechimi

$$y(x)=Ax^{-(1-\alpha)(1-\beta)}E_{\alpha,1-(1-\alpha)(1-\beta)}(\lambda x^{\alpha})+y(x_0)\int_0^x(x-t)^{\alpha-1}E_{\alpha,\alpha}\left[\lambda(x-t)^{\alpha}\right]dt\qquad(5)$$

ko'rinishda aniqlanadi.

(5) formuladagi integralni hisoblab,uni

$$y(x)=Ax^{-(1-\alpha)(1-\beta)}E_{\alpha,1-(1-\alpha)(1-\beta)}(\lambda x^{\alpha})+y(x_0)x^{\alpha}E_{\alpha,\alpha+1}(\lambda x^{\alpha})\qquad(6)$$

ko'rinishda yozib olamiz

(6) formulada $x=x_0$ deb, $y(x_0)$ ni

$$y(x_0)=\frac{Ax_0^{-(1-\alpha)(1-\beta)}E_{\alpha,1-(1-\alpha)(1-\beta)}(\lambda x_0^{\alpha})}{1-x_0^{\alpha}E_{\alpha,\alpha+1}(\lambda x_0^{\alpha})}\qquad(7)$$

ko'rinishda topamiz.

(7) ni (6) ga qo'yib, A masalaning yechimini

$$y(x)=Ax^{-(1-\alpha)(1-\beta)}E_{\alpha,1-(1-\alpha)(1-\beta)}(\lambda x^{\alpha})+\frac{Ax_0^{-(1-\alpha)(1-\beta)}E_{\alpha,1-(1-\alpha)(1-\beta)}(\lambda x_0^{\alpha})}{1-x_0^{\alpha}E_{\alpha,\alpha+1}(\lambda x_0^{\alpha})}E_{\alpha,\alpha+1}(\lambda x^{\alpha})\qquad(8)$$

ko'rinishda topamiz.

1-teorema. Agar $x_0^{\alpha}E_{\alpha,\alpha+1}(\lambda x_0^{\alpha})\neq1$ bo'lsa,u holda A masala yagona yechimga ega bo'lib, u (8) formula bilan aniqlanadi.

Endi (1) tenglamaning o'rniga

$$D_{0x}^{\alpha,\beta}y(x)-\lambda y(x)=D_{0x_0}^{\alpha}y(x_0)\qquad(9)$$

tenglamani (0,1) oraliqda qaraylik, bu yerda $D_{0x}^{\alpha}y(x)$-Riman-Liuvill ma'nosidagi kasr tartibli operatori,

$$D_{0x}^{\alpha}y(x)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dx}\int_0^x (x-t)^{-\alpha}y(t)dt.$$

B **masala.** (1) tenglamani o‘rniga (9) tenglamani A masalaning shartlarini bajaruvchi $y(x)$ funksiya topilsin.

(4) formuladan foydalanib, B masalaning (3) shartni qanoatlantiruvchi yechimini

$$y(x)=Ax^{-(1-\alpha)(1-\beta)}E_{\alpha,1-(1-\alpha)(1-\beta)}(\lambda x^{\alpha})+x^{\alpha}E_{\alpha,\alpha+1}(\lambda x^{\alpha})D_{0x_0}^{\alpha}y(x_0) \qquad (10)$$

ko‘rinishda yozib olamiz.

(10) formulaga D_{0x}^{α} ni ta'sir ettirib,

$$D_{0x}^{\alpha}y(x)=AD_{0x}^{\alpha}[x^{-(1-\alpha)(1-\beta)}E_{\alpha,1-(1-\alpha)(1-\beta)}(\lambda x^{\alpha})]+D_{0x}^{\alpha}[x^{\alpha}E_{\alpha,\alpha+1}(\lambda x^{\alpha})]D_{0x_0}^{\alpha}y(x_0) \qquad (11)$$

tenglikni hosil qilamiz

Ba'zi hisoblashlarni amalga oshirib,

$$D_{0x}^{\alpha}y(x)=Ax^{\beta(1-\alpha)-1}E_{\alpha,1-(1-\alpha)(1-\beta)-\alpha}(\lambda x^{\alpha})+E_{\alpha,1}(\lambda x^{\alpha})D_{0x_0}^{\alpha}y(x_0) \qquad (12)$$

(12) formulada $x=x_0$ deb, $D_{0x_0}^{\alpha}y(x_0)$ ni

$$D_{0x_0}^{\alpha}y(x_0)=\frac{Ax_0^{\beta(1-\alpha)-1}E_{\alpha,\beta(1-\alpha)}(\lambda x_0^{\alpha})}{1-E_{\alpha,1}(\lambda x_0^{\alpha})} \qquad (13)$$

ko‘rinishda topamiz.

(13) ni (10) ga qo‘yib, B masalaning yechimini

$$y(x)=Ax^{-(1-\alpha)(1-\beta)}E_{\alpha,1-(1-\alpha)(1-\beta)}(\lambda x^{\alpha})+\frac{Ax_0^{\beta(1-\alpha)-1}E_{\alpha,\beta(1-\alpha)}(\lambda x_0^{\alpha})}{1-E_{\alpha,1}(\lambda x_0^{\alpha})}x^{\alpha}E_{\alpha,\alpha+1}(\lambda x^{\alpha}) \qquad (14)$$

ko‘rinishda topamiz.

2-teorema. Agar $E_{\alpha,1}(\lambda x_0^{\alpha})\neq 1$ bo‘lsa, u holda B masala yagona yechimga ega bo‘lib, u (14) formula bilan aniqlanadi.

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