TO'RTINCHI TARTIBLI ODDIY DIFFERENSIAL TENGLAMA UCHUN NOLOKAL SHARTLI TESKARI MASALALAR

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Annotatsiya: *Ushbu maqolada to'rtinchi tartibli oddiy differensial tenglama uchun nolokal shartli teskari masalalar o`rganilgan. Bu masalalar yechimlari Grin funksiyalari usuli yordamida topilgan.*

Kalit so`zlar: *to'rtinchi tartibli oddiy differensial tenglama, nolokal shartli, teskari masala, Grin funksiyasi.*

НЕЛОКАЛЬНЫЕ УСЛОВНЫЕ ОБРАТНЫЕ ЗАДАЧИ ДЛЯ ОБЫКНОВЕННЫХ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ ЧЕТВЁРТОГО ПОРЯДКА

Аннотация: *В данной статье изучаются нелокальные условные обратные задачи для обыкновенных дифференциальных уравнений четвертого порядка. Решения этих задач были найдены с помощью метода функций Грина.*

Ключевые слова: *обыкновенное дифференциальное уравнение четвертого порядка, нелокальное условное уравнение, обратная задача, функция Грина.*

NONLOCAL CONDITIONAL INVERSE PROBLEMS FOR FOURTH ORDER ORDINARY DIFFERENTIAL EQUATIONS

Abstract: *In this article, nonlocal conditional inverse problems for fourth-order ordinary differential equations are studied. The solutions of these problems were found using the method of Green's functions.*

Keywords: *fourth - order ordinary differential equation, nonlocal conditional, inverse problem, Green's function.*

So'ngi vaqtlarda noma'lum manbali differensial tengalamalar bilan shug'ullanishga bo'lgan qiziqish ortib bormoqda. Bunga sabab ko'plab issiqlik tarqalish va diffuziya jarayonlarini matematik modelini tuzish noma'lum manbali differensial tenglama uchun qo'yiladigan masalalarga keltiriladi. Odatda, bunday turdagi masalalar teskari masala deb yuritiladi. Xususiy hosilali va oddiy differensial tenglamalar uchun teskari masalalar ko'plab tadqiqotchilar tomonidan o'rganilgan (masalan, ushbu [1]–[4] ishlarga qaralsin). Ammo yuqori tartibli tenglamalar uchun teskari masalalar kam o'rganilgan. Shu sababdan biz ushbu ishda to'rtinchi tartibli oddiy differensial tenglama uchun bir teskari masalani bir qiymatli yechilishini ko'rsatamiz.

(0, 1) oraliqda ushbu

$$y^{(4)}(x) = kf(x) \qquad (1)$$

to'rtinchi tartibli oddiy differensial tenglamani qaraylik, bu yerda $y(x)-$noma'lum funksiya; k nom'lum son, $f(x)-$ berilgan uzluksiz funksiya.

T_{pq} **masala.** Shunday $y(x)$ funksiya va k son topilsinki, u quyidagi xossalarga ega bo`lsin:

1) $y(x)\in C^3[0,1]\cap C^4(0,1)$ sinfga kirsin;

2) $y(x)$ funksiya (0,1) oraliqda (1) tenglamani qanoatlantirsin;

3) $[0,1]$ kesmada esa

$$py(0)=qy(1),\ y'(0)=0,\ y''(1)=0,\ qy'''(0)=py'''(1) \qquad (2)$$

chegaraviy va

$$y(1)=ay(\xi)+b \qquad (3)$$

nolokal shartni qanoatlantirsin, bu yerda a,b va ξ o`zgarmas haqiqiy sonlar bo`lib, $0 < \xi < 1$.

Odatda (3), shartni Bitsadze - Samariskiy tipidagi shart deyiladi.

Takidlash joizki, $p=0$ yoki $q=0$ bo'lgan xoli [5] ishda ko'rilgan.

T_{pq} masalada k sonini vaqtingcha ma'lum deb qarab, (1) tenglamani va (2) chegaraviy shartlardan foydalanib, uni Grin funksiyalari usuli bilan yechamiz. $\{(1),(2)\}$ masalaning Grin funksiyasi

$$G(x,s)\begin{cases}\dfrac{1}{6(p-q)}\left[x^2(-px+3(p-q)s+3)+q(3-s)s^2+\dfrac{5q}{(p-q)}\right], x<s;\\ \dfrac{1}{6(p-q)}\left[s^2(-ps+3(p-q)x+3)+q(3-x)x^2+\dfrac{5q}{(p-q)}\right], x>s,\end{cases} \qquad (4)$$

ko'rinishda aniqlanadi.

Topilgan Grin funksiyasi quyidagi xossalarga ega:

1^0. $G(x,s)$, $G_x(x,s)$ va $G_{xx}(x,s)$ funksiyalar s ning $(0,1)$ oraliqdagi barcha qiymatlarida x argumenti bo'yicha uzluksiz;

2^0. $G(x,s)$ funksiya (2) chegaraviy shartlarni bajaradi;

3^0. $G_{xxx}(x,s)$ hosila x ning $(0,s)$ va $(s,1)$ oraliqdagi barcha qiymatlarda uzluksiz bo'lib, $x=s$ nuqtada birinchi tur uzulishga ega va uning sakrashi 1 ga teng, ya'ni

$$\left[G_{xxx}(s+0,s)-G_{xxx}(s-0,s)\right]=1$$

yoki $$\left[G_{xxx}(s,s+0)-G_{xxx}(s,s-0)\right]=-1$$

4^0. $G(x,s)$ funksiya $(0,s)$ va $(s,1)$ oraliqda $G_{xxxx}(x,s)=0$ tenglamani qanoatlantiradi.

Gilbert teoremasiga ko`ra

$$y(x)=k\int_0^1 G(x,s)f(s)ds \qquad (5)$$

tenglikni yozib olamiz.

Endi k sonini topish maqsadida (5) formulani (3) shartga bo'ysundirib, ba'zi soddalashtirishlarni amalga oshirib,

$$k=\frac{b}{\int_0^1 G(1,s)f(s)ds-a\int_0^1 G(\xi,s)f(s)ds} \qquad (6)$$

tenglikni hosil qilamiz.

(6) formulani (5) formulaga qo‘yib, $y(x)$ funksiyani

$$y(x)=b\left[\int_0^1 G(1,s)f(s)ds-a\int_0^1 G(\xi,s)f(s)ds\right]^{-1}\int_0^1 G(1,s)f(s)ds \qquad (7)$$

ko‘rinishda aniqlaymiz.

1-teorema. Agar $\int_0^1 G(1,s)f(s)ds\neq a\int_0^1 G(\xi,s)f(s)ds$ bo‘lsa, u holda T_{pq} masala yagona yechimga ega bo‘ladi va u (6) va (7) formulalar bilan aniqlanadi.

1-izoh. Agar $\int_0^1 G(1,s)f(s)ds=a\int_0^1 G(\xi,s)f(s)ds$ va $b=0$ bo‘lsa, u holda T_{pq} masala cheksiz ko‘p yechimga ega bo‘ladi.

2-izoh. Agar $\int_0^1 G(1,s)f(s)ds=a\int_0^1 G(\xi,s)f(s)ds$ va $b\neq 0$ bo`lsa, u holda T_{pq} masala yechimga ega bo‘lmaydi.

T_{pq}^{n} **masala.** Shunday $y(x)$ funksiya k soni topilsinki, u T_{pq} masalaning barcha shartlari va (3) shartning o‘rniga

$$\alpha_1 y(\xi_1)+\alpha_2 y(\xi_2)+...+\alpha_n y(\xi_n)=b_1 \qquad (8)$$

nolokal shartni qanoatlantirsin, $\alpha_1,\alpha_2...\alpha_n$, $\xi_1,\xi_2...\xi_n$, b_1 o‘zgarmas haqiqiy sonlar.

T_{pq}^{n} masalani (2) shartni qanoatlantiruvchi yechimi (5) formula aniqlagani uchun (8) shartga qo‘yib, ba’zi soddalashtirishni bajarib

$$k=b_1\left[\alpha_1\int_0^1 G(\xi_1,s)f(s)ds+\alpha_2\int_0^1 G(\xi_2,s)f(s)ds+...+\alpha_n\int_0^1 G(\xi_n,s)f(s)ds\right]^{-1} \qquad (9)$$

tenglikni hosil qilamiz.

(9) formulani (5) formulaga qo‘yib, T_{pq}^{n} masalani yechimini

$$y(x)=b_1\left[\alpha_1\int_0^1 G(\xi_1,s)f(s)ds+\alpha_2\int_0^1 G(\xi_2,s)f(s)ds+...+\alpha_n\int_0^1 G(\xi_n,s)f(s)ds\right]^{-1}\int_0^1 G(x,s)f(s)ds \qquad (10)$$

ko‘rinishda aniqlaymiz.

2-teorema. Agar $\alpha_1\int_0^1 G(\xi_1,s)f(s)ds+\alpha_2\int_0^1 G(\xi_2,s)f(s)ds+...+\alpha_n\int_0^1 G(\xi_n,s)f(s)ds\neq 0$ bo`lsa, u holda T_{pq}^{n} masala yagona yechimga ega bo‘ladi va u (9), (10) formulalar bilan aniqlanadi.

3-izoh. Agar $\alpha_1\int_0^1 G(\xi_1,s)f(s)ds+\alpha_2\int_0^1 G(\xi_2,s)f(s)ds+...+\alpha_n\int_0^1 G(\xi_n,s)f(s)ds=0$ va $b_1=0$ bo‘lsa, u holda T_{pq}^{n} masala cheksiz ko‘p yechimga ega bo‘ladi.

4-izoh. Agar $\alpha_1\int_0^1 G(\xi_1,s)f(s)ds+\alpha_2\int_0^1 G(\xi_2,s)f(s)ds+...+\alpha_n\int_0^1 G(\xi_n,s)f(s)ds=0$ va bo'lsa, u holda T_{pq}^n masala yechimga ega bo'lmaydi.

I_{pq} **masala.** Shunday $y(x)$ funksiya k soni topilsinki, u T_{pq} masalaning barcha shartlari va

$$\int_0^1 y(x)dx=b_2 \qquad (11)$$

integral shartni qanoatlantirsin, bu yerda b_2 o'zgarmas haqiqiy son.

I_{pq} masalani (2) shartni qanoatlantiruvchi yechimi (5) formula aniqlagani uchun (11) shartga qo'yib, ba'zi soddalashtirishni bajarib,

$$k=b_2\left[\int_0^1\int_0^1 G(x,s)f(s)dsdx\right]^{-1} \qquad (12)$$

tenglikni hosil qilamiz.

(12) formulani (5) formulaga qo`yib, $y(x)$ funksiyani

$$y(x)=b_2\left[\int_0^1\int_0^1 G(x,s)f(s)dsdx\right]^{-1}\int_0^1 G(x,s)f(s)ds \qquad (14)$$

formula bilan aniqlaymiz.

3-teorema. Agar $\int_0^1\int_0^1 G(x,s)f(s)dsdx\neq 0$ bo'lsa, u holda I_{pq} masala yagona yechimga ega bo'ladi va u (13),(14) formulalar bilan aniqlanadi.

1-izoh. Agar $\int_0^1\int_0^1 G(x,s)f(s)dsdx=0$ va $b_2=0$ bo'lsa, u holda I_{pq} masala cheksiz ko'p yechimga ega bo'ladi.

2-izoh. Agar $\int_0^1\int_0^1 G(x,s)f(s)dsdx=0$ va $b_2\neq 0$ bo'lsa, u holda I_{pq} masala yechimga ega bo'lmaydi.

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