CHEGARALANMAGAN SOHADA UCHINCHI TARTIBLI TENGLAMA UCHUN BIR CHEGARAVIY MASALA XAQIDA

G`oziyev Q.S
Farg'ona davlat universiteti
Ro'ziyeva Zarnigor Otabek qizi
Farg'ona davlat universiteti

Annotatsiya: *Mazkur maqolada chegaralanmagan sohada uchinchi tartibli tenglama uchun chegaraviy masala qaralgan bo'lib, masalaning yechimining yagonaligi integral energiya usulida, yechimning mavjudligi esa Fredgolm integral tenglamasi orqali ifodalangan.*

Kalit so'zlar*: Yuqori tartibli tenglama, yechimning yagonaligi, Laplas operatori, yechimning mavjudligi, Grin funksiyasi.*

Bu tipdagi tenglamalar uchun ustozlarimiz T.D.Jo'rayev [1], M.S.Salohiddinov [2] va ularning shogirdlari tomonidan chegaraviy masalalar qo'yilib, ularni o'rganish nazariyalari yaratilgan. Hozirgi kunda ikkinchi, uchinchi va yuqori tartibli tenglamalar uchun ko'plab mualliflar tomonidan chegaraviy masalalar tahlil etilgan[4,5,6]. Ushbu maqolada uchinchi tartibli tenglama uchun chegaralanmagan sohada chegaraviy masala o'rganilgan.

Masalaning qo'yilishi. Ushbu

$$\frac{\partial}{\partial y}\Delta U(x,y)+C(x,y)U(x,y)=0 \qquad (1)$$

tenglamaning

$$D=\{(x,y):0<x<\infty, 0<y<\infty\}$$

sohada regulyar bo'lgan, $\overline{D}$ sohada uzluksiz shunday U(x,y) yechimini topingki, u quyidagi chegaraviy shartlarni qanoatlantirsin:

$$U(x,y)\big|_{x=0}=\varphi_1(y), \qquad 0\le y<\infty, \qquad (2)$$

$$U(x,y)\big|_{y=0}=\varphi_2(x), \qquad 0\le x<\infty, \qquad (3)$$

$$U_y(x,y)\big|_{y=0}=\varphi_3(x), \qquad 0\le x<\infty, \qquad (4)$$

$$\lim_{R\to\infty}U_y(x,y)=0, \qquad R^2=x^2+y^2, \qquad x>0, y>0. \qquad (5)$$

Bu yerda $\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ - Laplas operatori, C(x,y) va $\varphi_i(i=\overline{1,3})$ - berilgan funksiyalar va

$$\varphi_1(0)=\varphi_2(0) \qquad (6)$$

kelishuv sharti o'rinli.

Ushbu ko'rinishda belgilash kiritib,

$$U_y=V \qquad (7)$$

(1) tenglamani quyidagi ko'rinishda yozish mumkin:

$$\Delta V + CU = 0 \tag{8}$$

(7) ga asosan (2) - (5) chegaraviy shartlardan foydalanib, (8) tenglama uchun quyidagi chegaraviy shartlarni olamiz:

$$V(x,y)\big|_{x=0} = \varphi_1^{'}(y), \qquad V(x,y)\big|_{y=0} = \varphi_3(x), \tag{9}$$

$$\lim_{R\to\infty} V(x,y) = 0, \qquad R^2 = x^2 + y^2, \qquad x>0, y>0. \tag{10}$$

Masala yechimining yagonaligi.

Teorema. Agar $R\to\infty$ da

1) $\left|C(x,y)\right| \le \dfrac{C_1}{R^\alpha}, \qquad 0<\alpha<1, \quad C_1 = const;$

2) $C_y(x,y) \ge 0, \qquad (x,y)\in D,$

bo‘lsa, u holda (1) – (5) masalaning bittadan ortiq yechimi mavjud emas.

Isbot. Masala yechimining yagonaligini isbotlash uchun (1) - (5) masalaga mos bir jinsli masalaning trivial yechimga ega ekanligini ko‘rsatamiz, ya’ni

$$\Delta\left(\frac{\partial}{\partial y}U\right) + CU = 0 \tag{1}$$

$$U(x,y)\big|_{x=0} = 0, \qquad U(x,y)\big|_{y=0} = 0, \qquad U_y(x,y)\big|_{y=0} = 0. \tag{11}$$

masalani qaraymiz. (1) va (11) masala (7) belgilashga ko‘ra quyidagi masalaga ekvivalent:

$$\Delta V + CU = 0, \tag{8}$$

$$V(x,y)\big|_{x=0} = 0, \qquad V(x,y)\big|_{y=0} = 0, \tag{12}$$

$$\lim_{R\to\infty} V(x,y) = 0, \qquad R^2 = x^2 + y^2, \qquad x>0, y>0. \tag{10}$$

Ushbu

$$D_R = \left\{(x,y) : x^2 + y^2 < R^2, x>0, y>0\right\},$$

$$\partial D_R = \left\{(x,y) : (x=0)\cup(y=0)\cup\sigma_R\right\},$$

$$\sigma_R = \left\{(x,y) : x^2 + y^2 = R^2, x>0, y>0\right\}$$

sohani qaraymiz.

(8) Laplas tenglamasini V(x,y) funksiyaga ko‘paytirib, D_R soha bo‘yicha integral olamiz:

$$\iint_{D_R} V(V_{xx} + V_{yy} + CU)dxdy = 0. \tag{13}$$

Mazkur ko‘paytmada ba’zi shakl almashtirishlarni bajarib, (10), (12) chegaraviy shartlardan foydalansak, (13) tenglik quyidagi

$$\iint_{D_R}\left[(V_x)^2 + (V_y)^2 + \frac{1}{2}C_y U^2\right]dxdy = 0. \tag{14}$$

ko‘rinishga keladi. Bundan,

1) Agar $C_y \ne 0$ bo‘lsa, u holda (14) dan $\overline{D}$ sohada U=0 ekanligi kelib chiqadi.

2) Agar $C_y = 0$ bo'lsa, u holda (14) dan $V_x = V_y = 0$ tenglikni olamiz, bu tengliklardan

$V = const$ ekanligi kelib chiqadi. Ammo bir jinsli (12) shartga asosan $\overline{D}$ da $V = 0$ yoki (7) belgilashga ko'ra $U_y = 0$ bo'ladi. Ma'lumki, bu tenglamaning umumiy yechimi

$$U = \overline{\phi}(x) \qquad (15)$$

ko'rinishda bo'ladi. Bu yerda $\overline{\phi}(x)$ - ixtiyoriy noma'lum funksiya. (11) chegaraviy shartlarning biridan foydalansak, (15) tenglikdagi $\overline{\phi}(x)$ funksiya aynan nolga teng bo'lib $U(x, y) = 0,\ (x, y) \in \overline{D}$ da $U_y = 0$ tenglama trivial yechimga ega ekanligini topamiz.

Masala yechimining mavjudligi.

2-teorema. Agar

1) $\varphi_1(y), \varphi_2(x), \varphi_3(x)$ funksiyalar uzluksiz va

2) $\left|\varphi_1'\right| \le \frac{C_1}{y^2}, \quad y \to \infty, \ |\varphi_2| \le \frac{C_2}{x^2}, \ |\varphi_3| \le \frac{C_3}{x^2}, \ x \to \infty$ shartlarni qanoatlantirsa,

u holda (1) - (5) masalaning yechimi mavjud.

Isbot. Yechimning mavjudligini isbotlash uchun Grin funksiyalari [3] usulidan foydalanamiz. U holda (8), (10), (12) masalaning yechimini ifodalovchi quyidagi formulaga kelamiz:

$$V(x, y) = \int_0^\infty G_\xi\left(x, y; 0, \eta\right)\varphi_1'(\eta)d\eta - \int_0^\infty G_\eta\left(x, y; \xi, 0\right)\varphi_3(\xi)d\xi - \\ -\iint_D G_\eta(x, y; \xi, \eta)C(\xi, \eta)U(\xi, \eta)d\xi d\eta \qquad (16)$$

bu yerda $G(z, \tau) = \frac{1}{2\pi}\ln\left|\frac{z^2 - \tau^{-2}}{z^2 - \tau^2}\right|;$: z=x+iy, $\tau = \xi + i\eta$. yoki

$$G(z, \tau) = -\frac{1}{2\pi}\ln|z - \tau| + g\left(x, y; \xi, \eta\right) = -\frac{1}{2\pi}\ln\sqrt{\left(x - \xi\right)^2 + \left(y - \eta\right)^2} + \\ + g\left(x, y; \xi, \eta\right) = -\frac{1}{2\pi}\ln r + g\left(x, y; \xi, \eta\right) \qquad (17)$$

Grin funksiyasining (17) ko'rinishidan foydalansak (16) ifoda quydagicha yoziladi:

$$V(x, y) = \frac{1}{\pi}\int_0^\infty\left(\frac{x}{x^2 + (y - \eta)^2} - \frac{x}{x^2 + (y + \eta)^2}\right)\varphi_1'(\eta)d\eta - \\ -\frac{1}{\pi}\int_0^\infty\left(\frac{y}{\left(x - \xi\right)^2 + y^2} - \frac{y}{\left(x + \xi\right)^2 + y^2}\right)\varphi_3(\xi)d\xi - \iint_{D_R^\xi} GCUd\xi d\eta \qquad (18)$$

Endi (7) tenglamani (3) shartini qanoatlantiruvchi yechimini qidiramiz. Buning uchun (7) tenglamani [0, x] segmentda integrallab, (18) dan foydalansak,

$$U(x,y)=\frac{1}{\pi}\int_0^y\left[\int_0^\infty\left(\frac{x}{x^2+(t-\eta)^2}-\frac{x}{x^2+(t+\eta)^2}\right)\varphi_1'(\eta)d\eta\right]dt-$$
$$-\frac{1}{\pi}\int_0^\infty\left(\frac{t}{(x-\xi)^2+t^2}-\frac{t}{(x+\xi)^2+t^2}\right)\varphi_3(\xi)d\xi-\iint_D GCUd\xi d\eta+\varphi_1(x) \quad (19)$$

(19) tenglikda integrallarni hisoblash natijasida quyidagi

$$U(x,y)+\iint_D K(x,y;\xi,\eta)U(\xi,\eta)d\xi d\eta=F(x,y) \quad (20)$$

ko'rinishdagi tenglamaga kelamiz. Bu yerda

$$K(x,y;\xi,\eta)=C(\xi,\eta)\int_0^y G(x,t;\xi,\eta)dt$$

$$F(x,y)=-\int_0^\infty G_1(x,y;0,\eta)\varphi_1'(\eta)d\eta+\int_0^\infty G_2(x,y;\xi,0)\varphi_3(\xi)d\xi+\varphi_1(x),$$

$$G_1(x,y;0,\eta)=arctg\frac{x-\eta}{x}-arctg\frac{x+\eta}{x},$$

$$G(x,y;\xi,0)=\frac{1}{2}\left[\ln(x^2+(x-\xi)^2)-\ln(x^2+(x+\xi)^2)+2\ln\frac{x+\xi}{x-\xi}\right].$$

(20) tenglama Fredgolmning ikkinchi tur integral tenglamasi bo'lib, yechimning yagonaligi mavjudlik teoremasidan kelib chiqadi.

Mavjudlik teoremasi shartlari bajarilganda U(x, y) funksiya masalada qo'yilgan barcha shartlarni bajaradi. Teorema isbotlandi.

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