

**CHEGARALANMAGAN SOHADA UCHINCHI TARTIBLI TENGLAMA
UCHUN BIR CHEGARAVIY MASALA XAQIDA**

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Annotatsiya: *Mazkur maqolada chegaralanmagan sohada uchinchi tartibli tenglama uchun chegaraviy masala qaralgan bo'lib, masalaning yechimining yagonaligi integral energiya usulida, yechimning mavjudligi esa Fredgolm integral tenglamasi orqali ifodalangan.*

Kalit so'zlar: *Yuqori tartibli tenglama, yechimning yagonaligi, Laplas operatori, yechimning mavjudligi, Grin funksiyasi.*

Bu tipdagi tenglamalar uchun ustozlarimiz T.D.Jo'rayev [1], M.S.Salohiddinov [2] va ularning shogirdlari tomonidan chegaraviy masalalar qo'yilib, ularni o'rganish nazariyalari yaratilgan. Hozirgi kunda ikkinchi, uchinchi va yuqori tartibli tenglamalar uchun ko'plab mualliflar tomonidan chegaraviy masalalar tahlil etilgan[4,5,6]. Ushbu maqolada uchinchi tartibli tenglama uchun chegaralanmagan sohada chegaraviy masala o'rganilgan.

Masalaning qo'yilishi. Ushbu

$$\frac{\partial}{\partial y} \Delta U(x, y) + C(x, y)U(x, y) = 0 \quad (1)$$

tenglamaning

$$D = \{(x, y) : 0 < x < \infty, 0 < y < \infty\}$$

sohada regulyar bo'lgan, \bar{D} sohada uzluksiz shunday $U(x, y)$ yechimini topingki, u quyidagi chegaraviy shartlarni qanoatlantirsin:

$$U(x, y)|_{x=0} = \varphi_1(y), \quad 0 \leq y < \infty, \quad (2)$$

$$U(x, y)|_{y=0} = \varphi_2(x), \quad 0 \leq x < \infty, \quad (3)$$

$$U_y(x, y)|_{y=0} = \varphi_3(x), \quad 0 \leq x < \infty, \quad (4)$$

$$\lim_{R \rightarrow \infty} U_y(x, y) = 0, \quad R^2 = x^2 + y^2, \quad x > 0, y > 0. \quad (5)$$

Bu yerda $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ - Laplas operatori, $C(x, y)$ va $\varphi_i (i = \overline{1, 3})$ - berilgan

funksiyalar va

$$\varphi_1(0) = \varphi_2(0) \quad (6)$$

kelishuv sharti o'rinli.

Ushbu ko'rinishda belgilash kiritib,

$$U_y = V \quad (7)$$

(1) tenglamani quyidagi ko'rinishda yozish mumkin:

$$\Delta V + CU = 0 \tag{8}$$

(7) ga asosan (2) - (5) chegaraviy shartlardan foydalanib, (8) tenglama uchun quyidagi chegaraviy shartlarni olamiz:

$$V(x, y)|_{x=0} = \varphi_1(y), \quad V(x, y)|_{y=0} = \varphi_3(x), \tag{9}$$

$$\lim_{R \rightarrow \infty} V(x, y) = 0, \quad R^2 = x^2 + y^2, \quad x > 0, y > 0. \tag{10}$$

Masala yechimining yagonaligi.

Teorema. Agar $R \rightarrow \infty$ da

$$1) |C(x, y)| \leq \frac{C_1}{R^\alpha}, \quad 0 < \alpha < 1, \quad C_1 = const;$$

$$2) C_y(x, y) \geq 0, \quad (x, y) \in D,$$

bo‘lsa, u holda (1) - (5) masalaning bittadan ortiq yechimi mavjud emas.

Isbot. Masala yechimining yagonaligini isbotlash uchun (1) - (5) masalaga mos bir jinsli masalaning trivial yechimga ega ekanligini ko‘rsatamiz, ya’ni

$$\Delta \left(\frac{\partial}{\partial y} U \right) + CU = 0 \tag{1}$$

$$U(x, y)|_{x=0} = 0, \quad U(x, y)|_{y=0} = 0, \quad U_y(x, y)|_{y=0} = 0. \tag{11}$$

masalani qaraymiz. (1) va (11) masala (7) belgilashga ko‘ra quyidagi masalaga ekvivalent:

$$\Delta V + CU = 0, \tag{8}$$

$$V(x, y)|_{x=0} = 0, \quad V(x, y)|_{y=0} = 0, \tag{12}$$

$$\lim_{R \rightarrow \infty} V(x, y) = 0, \quad R^2 = x^2 + y^2, \quad x > 0, y > 0. \tag{10}$$

Ushbu

$$D_R = \{(x, y) : x^2 + y^2 < R^2, x > 0, y > 0\},$$

$$\partial D_R = \{(x, y) : (x=0) \cup (y=0) \cup \sigma_R\},$$

$$\sigma_R = \{(x, y) : x^2 + y^2 = R^2, x > 0, y > 0\}$$

sohani qaraymiz.

(8) Laplas tenglamasini $V(x, y)$ funksiyaga ko‘paytirib, D_R soha bo‘yicha integral olamiz:

$$\iint_{D_R} V(V_{xx} + V_{yy} + CU) dx dy = 0. \tag{13}$$

Mazkur ko‘paytmada ba’zi shakl almashtirishlarni bajarib, (10), (12) chegaraviy shartlardan foydalansak, (13) tenglik quyidagi

$$\iint_{D_R} \left[(V_x)^2 + (V_y)^2 + \frac{1}{2} C_y U^2 \right] dx dy = 0. \tag{14}$$

ko‘rinishga keladi. Bundan,

1) Agar $C_y \neq 0$ bo‘lsa, u holda (14) dan \overline{D} sohada $U=0$ ekanligi kelib chiqadi.

2) Agar $C_y = 0$ bo'lsa, u holda (14) dan $V_x = V_y = 0$ tenglikni olamiz, bu tengliklardan

$V = const$ ekanligi kelib chiqadi. Ammo bir jinsli (12) shartga asosan \bar{D} da $V = 0$ yoki (7) belgilashga ko'ra $U_y = 0$ bo'ladi. Ma'lumki, bu tenglamaning umumiy yechimi

$$U = \bar{\phi}(x) \tag{15}$$

ko'rinishda bo'ladi. Bu yerda $\bar{\phi}(x)$ - ixtiyoriy noma'lum funksiya. (11) chegaraviy shartlarning biridan foydalansak, (15) tenglikdagi $\bar{\phi}(x)$ funksiya aynan nolga teng bo'lib $U(x, y) = 0$, $(x, y) \in \bar{D}$ da $U_y = 0$ tenglama trivial yechimga ega ekanligini topamiz.

Masala yechimining mavjudligi.

2-teorema. Agar

1) $\varphi_1(y), \varphi_2(x), \varphi_3(x)$ funksiyalar uzluksiz va

2) $|\varphi_1'| \leq \frac{C_1}{y^2}$, $y \rightarrow \infty$, $|\varphi_2| \leq \frac{C_2}{x^2}$, $|\varphi_3| \leq \frac{C_3}{x^2}$, $x \rightarrow \infty$ shartlarni qanoatlantirsa,

u holda (1) - (5) masalaning yechimi mavjud.

Isbot. Yechimning mavjudligini isbotlash uchun Grin funksiyalari [3] usulidan foydalanamiz. U holda (8), (10), (12) masalaning yechimini ifodalovchi quyidagi formulaga kelamiz:

$$V(x, y) = \int_0^\infty G_\xi(x, y; 0, \eta) \varphi_1'(\eta) d\eta - \int_0^\infty G_\eta(x, y; \xi, 0) \varphi_3(\xi) d\xi - \iint_D G_\eta(x, y; \xi, \eta) C(\xi, \eta) U(\xi, \eta) d\xi d\eta \tag{16}$$

bu yerda $G(z, \tau) = \frac{1}{2\pi} \ln \left| \frac{z^2 - \tau^{-2}}{z^2 - \tau^2} \right|$; $z = x + iy$, $\tau = \xi + i\eta$. yoki

$$G(z, \tau) = -\frac{1}{2\pi} \ln |z - \tau| + g(x, y; \xi, \eta) = -\frac{1}{2\pi} \ln \sqrt{(x - \xi)^2 + (y - \eta)^2} + g(x, y; \xi, \eta) = -\frac{1}{2\pi} \ln r + g(x, y; \xi, \eta) \tag{17}$$

Grin funksiyasining (17) ko'rinishidan foydalansak (16) ifoda quydagicha yoziladi:

$$V(x, y) = \frac{1}{\pi} \int_0^\infty \left(\frac{x}{x^2 + (y - \eta)^2} - \frac{x}{x^2 + (y + \eta)^2} \right) \varphi_1'(\eta) d\eta - \frac{1}{\pi} \int_0^\infty \left(\frac{y}{(x - \xi)^2 + y^2} - \frac{y}{(x + \xi)^2 + y^2} \right) \varphi_3(\xi) d\xi - \iint_{D_R^*} G C U d\xi d\eta \tag{18}$$

Endi (7) tenglamani (3) shartini qanoatlantiruvchi yechimini qidiramiz. Buning uchun (7) tenglamani $[0, x]$ segmentda integrallab, (18) dan foydalansak,

$$U(x, y) = \frac{1}{\pi} \int_0^y \left[\int_0^\infty \left(\frac{x}{x^2 + (t-\eta)^2} - \frac{x}{x^2 + (t+\eta)^2} \right) \varphi_1'(\eta) d\eta \right] dt - \frac{1}{\pi} \int_0^\infty \left(\frac{t}{(x-\xi)^2 + t^2} - \frac{t}{(x+\xi)^2 + t^2} \right) \varphi_3(\xi) d\xi - \iint_D GCU d\xi d\eta + \varphi_1(x) \quad (19)$$

(19) tenglikda integrallarni hisoblash natijasida quyidagi

$$U(x, y) + \iint_D K(x, y; \xi, \eta) U(\xi, \eta) d\xi d\eta = F(x, y) \quad (20)$$

ko'rinishdagi tenglamaga kelamiz. Bu yerda

$$K(x, y; \xi, \eta) = C(\xi, \eta) \int_0^y G(x, t; \xi, \eta) dt$$

$$F(x, y) = - \int_0^\infty G_1(x, y; 0, \eta) \varphi_1'(\eta) d\eta + \int_0^\infty G_2(x, y; \xi, 0) \varphi_3(\xi) d\xi + \varphi_1(x),$$

$$G_1(x, y; 0, \eta) = \operatorname{arctg} \frac{x-\eta}{x} - \operatorname{arctg} \frac{x+\eta}{x},$$

$$G(x, y; \xi, 0) = \frac{1}{2} \left[\ln(x^2 + (x-\xi)^2) - \ln(x^2 + (x+\xi)^2) + 2 \ln \frac{x+\xi}{x-\xi} \right].$$

(20) tenglama Fredgolmning ikkinchi tur integral tenglamasi bo'lib, yechimning yagonaligi mavjudlik teoremasidan kelib chiqadi.

Mavjudlik teoremasi shartlari bajarilganda $U(x, y)$ funksiya masalada qo'yilgan barcha shartlarni bajaradi. Teorema isbotlandi.

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